HEAT TRANSFER DURING LAMINAR FLOW IN A VERTICAL ANNULAR CHANNEL WITH A CONSTANT TEMPERATURE OF THE OUTER WALL

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Developed flows during combined convection in a vertical concentric annular channel are discussed. The problem is solved for descending and ascending flows. A dimensionless equation is obtained for calculating the value of the heat-transfer coefficient averaged over the length of the channel.

A vertical annular channel is a natural and very efficient shape for constraining the flow of a coolant with a developed heat-transfer surface. The use of such channels permits a rather simple construction of compact straight-through or countercurrent devices. Such schemes are used in air and liquid ground freezing columns employed in industrial and hydraulic engineering structures in Siberia and the far north.

Many papers [1-10] have appeared on combined convection in vertical channels. These include papers in which an analytic solution of the problem is obtained in the so-called "shell" approximation [1] and studies based on various integral [2-4] and finite-difference [6-10] methods. Only a few papers, however, treat the problem of combined convection in finite-length annular channels. The most detailed study of this problem was made by Beck [5] and also in [3, 4], in which the Pigford integral method [2] was extended to the case of an annular tube.

We use an explicit finite-difference method to solve the problem of combined convection in a vertical concentric annular channel for free and forced convection in the same and opposite directions.

The problem is solved under the assumption that the inner tube of the channel is thermally insulated, the temperature of the nonadiabatic outer wall is constant, and the velocity and temperature profiles in the entrance cross section are uniform. In addition, it is assumed that the physical properties of the liquid, except the density, are constant. The variation of the density with the temperature is assumed linear and is taken into account only in the term in the equation of motion which describes the buoyant force.

The dimensionless hydrodynamics and heat-transfer equations describing the axisymmetric flow of a liquid in a vertical annular channel and the appropriate boundary conditions have the form [11]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial r}+\frac{v}{r}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=B_1 t-\frac{\partial p}{\partial x}+\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{2}$$

$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial r}=\frac{1}{\mathrm{Pr}}\left(\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial t}{\partial r}\right), \tag{3}$$

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$$\int_{r_1}^{r_2} urdr = \frac{r_2^2 - r_1^2}{2} u_0, \tag{4}$$

$$\begin{gathered} r_1 < r < r_2,\ x = 0,\ u = u_0,\ v = p = t = 0, \\ r = r_1,\ x \geqslant 0,\ u = v = 0,\ \partial t/\partial r = 0, \\ r = r_2,\ x \geqslant 0,\ u = v = 0,\ t = 1. \end{gathered} \tag{5}$$

The system of equations (1)-(4) with boundary conditions (5) was solved numerically by using an explicit finite-difference method which is a modification of the Bodoia-Osterle method [6] applied to an annular channel. The order of approximation of the difference scheme used is $O(\Delta x) + O(\Delta r^2)$. At each step the system of difference analogs of Eqs. (1)-(4) was solved in the following sequence: the energy equation, the momentum equation simultaneously with the condition of constant flow rate, and the equation of continuity.

The calculations were performed for free and forced convection in the same and opposite directions. In the calculational process the physical properties of the coolant, the channel geometry, the flow rate of the liquid, and the temperature difference between the nonadiabatic wall and the coolant were varied.

Figures 1 and 2 show the development of the dimensionless axial velocity profile and the dimensionless temperature profile for free and forced convection in the same and opposite directions. The value of A = Gr/Re has a major effect on the development of the velocity profile.

Calculations showed that for $|A| \leq 100$ the effect of free convection is vanishingly small and the velocity profile develops as in pure forced flow. For $|A| > 100$ the development of the velocity profile is significantly different from the corresponding isothermal case, as is clear from the figures.

A general qualitative characteristic can be obtained by considering the balance of forces responsible for the nature of the flow. Beginning with the entrance cross section both thermal and dynamic boundary layers develop simultaneously at the nonadiabatic outer wall. A buoyant force develops in the heated or cooled boundary layer of the coolant. So long as the volume of liquid in the thermal layer is small, the buoyant forces have little effect on the shape of the velocity profile, and the flow has a viscous-inertial character. The length of this portion depends on the Prandtl number. The smaller the Prandtl number, the more rapidly the thermal boundary layer develops, and the more rapidly the flow goes over into the regime where free convection predominates. As the thermal boundary layer increases in thickness the buoyant forces become comparable to or larger than the inertial and viscous forces. From this instant the velocity profile is strongly distorted: The liquid is either speeded up (Fig. 1a) or slowed down (Fig. 2a) at the wall depending on the direction of the heat flow. Simultaneously, as a consequence of the constant flow rate the velocity in the core decreases or increases, respectively. The viscous-gravitational flow region continues until the thermal boundary layer reaches the inner thermally insulated wall. Then the whole flow volume begins to get heated (Fig. 1b) or cooled (Fig. 2b) to the temperature of the outer wall, and the effect of free convection decreases. The flow reverts to the viscous-inertial regime until it is stabilized. The dimensions of the characteristic zones enumerated depend on the Prandtl number, A and the channel width.

Free convection affects the resistance as a consequence of the rearrangement of the velocity profile and friction losses. The hydraulic resistance coefficients were calculated from the pressure distribution, and the coefficient of friction was calculated from the velocity distribution.

The heat-transfer data are shown in Figs. 3 and 4. Figure 3 shows the variation of the local Nusselt number along the length of a channel of width K = 0.25 for various values of the Prandtl number and A.

It can be seen from Fig. 3 that for pure forced flow (A = 0) the dimensionless heat-transfer coefficient approaches a limiting value which depends on the width of the channel. The calculated values of the limiting Nusselt numbers agree with other data [11].

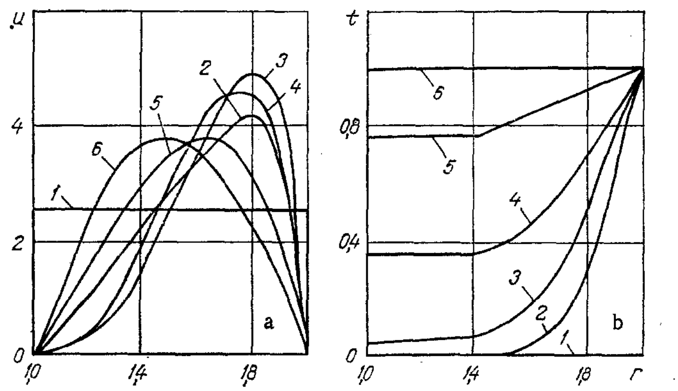

Fig. 1. Typical developing profiles of dimensionless axial velocity (a) and dimensionless temperature (b) for forced and free convection in the same direction when K = 0.5, Pr = 1, A = 1000; 1) x = 0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 1.0; 6) ∞.

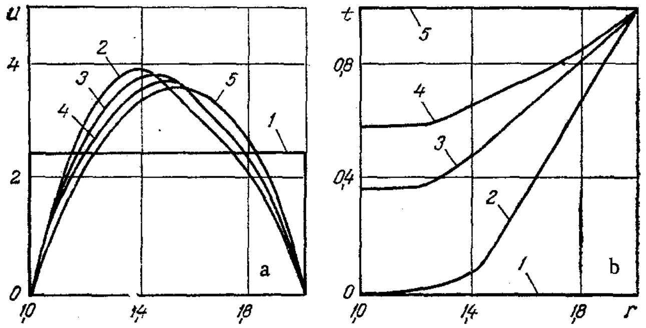

Fig. 2. Typical developing profiles of dimensionless axial velocity (a) and dimensionless temperature (b) for forced and free convection in opposite directions when K = 0.5, Pr = 1, A = −100; 1) x = 0; 2) 0.1; 3) 0.6; 4) 1.0; 5) ∞.

It can be seen from Fig. 3 that when free convection is superimposed on forced flow the heat-transfer curve has a local maximum at a certain distance from the entrance cross section. This behavior of the local Nusselt number when the directions of free and forced convection coincide has been noted experimentally [5] for an annular channel and theoretically [9, 10] for a circular tube.

This phenomenon can be accounted for in the following way. Close to the channel entrance free convection has almost no effect, and therefore the same relationships are observed as in forced flow; i.e., the heat transfer decreases.

Farther from the entrance cross section free convection begins to have an effect, since the boundary layer of the liquid is heated up and a buoyant force develops. The flow rate in the boundary layer is increased somewhat as a result of free convection, so that in spite of a decrease in density the flow rate through a cross section of the boundary layer exceeds the flow rate through that same cross section for a uniform velocity profile. Therefore, the displacement of liquid from the boundary layer into the flow core and the acceleration of the core, which are usual in forced convection, do not occur; on the contrary, the velocity in the flow core is decreased and the flow lines are diverted toward the wall. As this occurs liquid is displaced from the flow core into the boundary layer and there is a further acceleration of the flow at the wall, which leads to increased heat transfer as compared with forced flow.

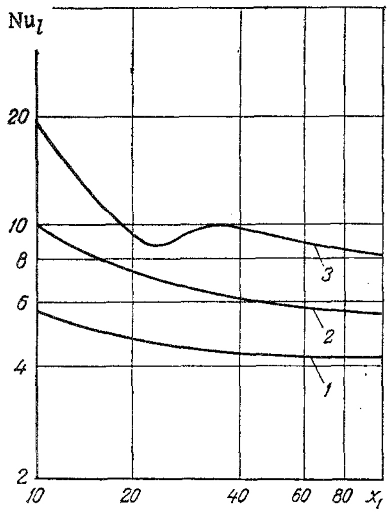

Fig. 3

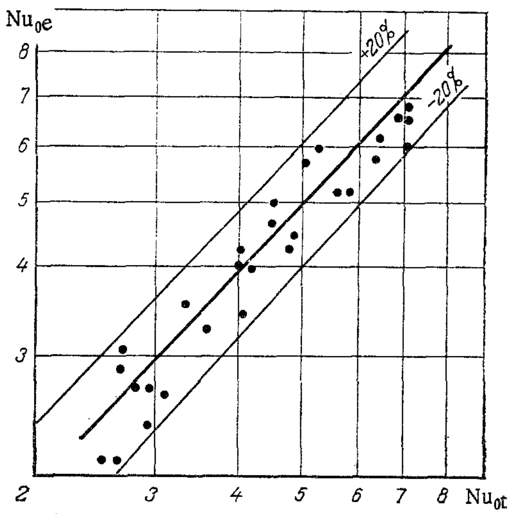

Fig. 4

Fig. 3. Variation of local Nusselt number along the length of an annular channel of width K = 0.25 when the directions of free and forced convection coincide. 1) Pr = 1, A = 0; 2) Pr = 10, A = 0; 3) Pr = 10, A = 100.

Fig. 4. Comparison of theory and experiment.

As the thermal boundary layer reaches the inner thermally insulated wall the temperature of the liquid begins to level out over the cross section of the channel, and heat transfer decreases toward its limiting value.

Calculations show that as the width of the channel is decreased, i.e., as K is increased, free convection has a smaller effect on heat transfer, and the maximum on the graph of the local Nusselt number becomes progressively flatter and less distinct.

Figure 4 compares the calculated local Nusselt numbers with the experimental local heat-transfer coefficients [12]. The theoretical and experimental results differ at most by ±20%.

The calculated Nusselt numbers averaged over the length of the channel are well approximated by the empirical relation

$$\mathrm{Nu}_{\mathrm{av}} = 2\mathrm{Pr}^{n}(1-K)^{1/16}\left(\frac{d_e}{L}\right)^{1/3}\left[\mathrm{Re}^{1/3} + 0.0136\cdot\exp\left(-\frac{1.23}{\mathrm{Pr}^2}\right)\mathrm{Gr}^{1/6}\left(\frac{\mathrm{Gr}}{\mathrm{Re}}\right)^{1/2}\right], \tag{6}$$

$$n = 0.27 + \frac{0.3}{\mathrm{Pr}} + 0.06K,$$

which takes account of the effect of free and forced convection, size, and the physical properties of the coolant.

Equation (6) is valid when free and forced convection are in the same direction for $1 \leq \mathrm{Pr} \leq 50$, $0 \leq \mathrm{Gr} \leq 10^6$, $100 \leq \mathrm{Re} \leq 2000$, and $10 \leq L/d_e \leq 200$.

NOTATION

$d_1 = 2r_1$, $d_2 = 2r_2$, diameters of inner and outer tubes; $d_e = 2R_e = d_2 - d_1$, equivalent diameter; L, length; $K = d_1/d_2$, dimensionless width of annular channel; P, pressure, P_0, hydrostatic pressure; $p = (P - P_0)R_e^4/\rho L^2\nu^2)$, dimensionless pressure of liquid; R, X, radial and axial coordinates; $r = R/R_e$, $x = X/L$, $x_1 = X/d_e$, dimensionless coordinates; T, temperature; T_0, entrance temperature of liquid; T_w, temperature of outer wall of channel; T_m, mean bulk temperature of liquid in cross section under consideration; $t = (T - T_0)/(T_w - T_0)$, dimensionless temperature; U, V, axial and radial velocity components; $v = VR_e/\nu$, $u = UR_e^2/\nu L$, dimensionless velocity components; u_0, dimensionless axial velocity at channel entrance; g, acceleration due to gravity; β, volume coefficient of expansion, λ, thermal conductivity; C_p, specific heat; ρ, density; ν, coefficient of kinematic viscosity; a, thermal diffusivity of liquid;

$Pr = \nu/a$, Prandtl number; $Re = Ud_e/\nu$, Reynolds number; $Gr = g\beta d_e^3 (T_w - T_0)/\nu^2$, Grashof number; $B_1 = Gr/(16d_e/L)$, parameter in momentum equation; α_0, local heat-transfer coefficient calculated from initial temperature head $(T_w - T_0)$; α_m, local heat-transfer coefficient calculated from local temperature head $(T_w - T_m)$; $Nu_0 = \alpha_0 d_e/\lambda$, $Nu_m = \alpha_m d_e/\lambda$, local Nusselt numbers; $Nu_{av} = \int_0^1 Nu_0 dx$; Nusselt number averaged over length of channel; Nu_{0t}, Nu_{0e}, calculated and experimental local Nusselt numbers.

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INTENSIFICATION OF HEAT EXCHANGE IN BUNDLES OF RODS LONGITUDINALLY BATHED BY GAS

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Functions for the calculation of heat exchange and hydraulic resistance in bundles of rods with artificial roughness are presented and the optimum dimensions and shape of the roughness are indicated.

The creation of high-intensity thermoenergetic devices requires the intensification of the processes of heat exchange. This problem is particularly urgent for devices having gas cooling, since gas coolants, while having a number of advantages over others (safety in operation, the possibility of use in atomic gas-turbine installations, etc.), have drawbacks connected with the low density, small heat capacity, and low coefficient of thermal conductivity.

Various means exist for the intensification of gaseous heat exchange. One of the simplest means is an increase in the velocity of the coolant, but the possibilities of this means are limited, since with an increase in the velocity of the coolant the resistance and the power for pumping the coolant grow simultaneously with the increase in the heat-transfer coefficient. Another means of intensification of heat exchange is based on the use of more

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